Pathria 2.1. A simple argument that requires some knowledge of canonical transformations uses the fact that the volume element transform by the determinant of of the Jacobian of the coordinate trans.

Suppose $Q_i \equiv Q_i(q_i, p_i)$, $P_i \equiv P_i(q_i, p_i)$ defines the canonical trans. Then with summation implied

$$dQ_i\, dQ^i\, dP_j\, dP^j = ||M||\, dq_i\, dq^i\, dp_j\, dp^j$$

where $|M|$ is the determinant of the Jacobian M.

Canonical transformations satisfy the symplectic relation (see goldstein 9.5)

$$M J M^T = J \quad \text{where} \quad J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}.$$

$$\Rightarrow |M|^2 = 1, \quad ||M|| = 1.$$

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